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THE $\phi \to \pi^+\pi^-$ AND ϕ RADIATIVE DECAYS WITHIN A CHIRAL UNITARY APPROACH

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ABSTRACT

We report on recent results on the decay of the ϕ into $\pi^+\pi^-$ and ϕ radiative decays into $\pi^0\pi^0\gamma$ and $\pi^0\eta\gamma$, which require the consideration of the final state interaction of a pair of mesons in a region inaccessible to Chiral Perturbation Theory. By using nonperturbative chiral unitary methods for the meson meson interaction we can obtain the corresponding decay widths and the results are compared with recent experimental data.

1 The $\phi \to \pi^+\pi^-$ decay

The ϕ decay into $\pi^+\pi^-$ is an example of isospin violation. The ϕ has isospin $I = 0$, spin $J = 1$, and hence it does not couple to the $\pi^+\pi^-$ system in the isospin limit, which implies the rule $I + J =$ even. The experimental situation on this decay is rather confusing. There are two older results whose central

values are very different but their quoted errors are so big that both were still compatible: The first one from [1] gives $BR = (1.94 + 1.03 - 0.81) \times 10^{-4}$. The second one from [2] provides $BR = (0.63 + 0.37 - 0.28) \times 10^{-4}$. Very recently two new, more precise, but conflicting results have been reported from the two experiments at the VEPP-2M in Novosibirsk: the CMD-2 Collaboration reports a value $BR = (2.20 \pm 0.25 \pm 0.20) \times 10^{-4}$ [3] whereas the SND Collaboration [4] obtains $BR = (0.71 \pm 0.11 \pm 0.09) \times 10^{-4}$.

Isospin violation has become a fashionable topic in Chiral Perturbation Theory (χPT) [5, 6] but the $\phi \to \pi\pi$ decay is however unreachable with plain χPT, since it involves the propagation of the pair of pions around 1 GeV, far away from the χPT applicability range.

Nevertheless, new nonperturbative schemes imposing unitarity and still using the chiral Lagrangians have emerged enlarging the convergence of the chiral expansion. In [7] the inverse amplitude method (IAM) is used in one channel and good results are obtained for the σ, ρ and K^* regions, amongst others, in $\pi\pi$ and πK scattering. In [8, 9] the method is generalized to include coupled channels and one is able to describe very well the meson-meson scattering and all the associated resonances up to about 1.2 GeV. A more general approach is used in [10] by means of the N/D method, in order to include the exchange of some preexisting resonances explicitly, which are then responsible for the values of the parameters of the fourth order chiral Lagrangian.

Here we shall follow the work [8] since it provides the most complete study of the different meson-meson scattering channels, including the mesonic resonances and their properties up to 1.2 GeV. In particular, this method yields a resonance in the $I = 0, J = 1$ channel, the ω_8 resonance, related to the ϕ, and this allows us to obtain the strong contribution to the $\phi \to \pi\pi$ decay. We also consider electromagnetic contributions at tree level which turn out to be dominant and were already considered in [11, 12].

In order to calculate the contribution of an intermediate photon to the $\phi \to \pi\pi$ decay, let us consider the effective Lagrangian for vector mesons presented in [13], which is written in terms of the SU(3) pseudoscalar meson matrix Φ and the antisymmetric vector tensor field $V_{\mu\nu}$ defined in [13]

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{i\, G_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle, \qquad (1)$$

where "$\langle\ \rangle$" indicates the SU(3) trace. In order to introduce the physical states

ϕ and ω, we assume ideal mixing between the ω_1 and ω_8 vector resonances and hence taking into account that the ω_1 does not couple to pairs of mesons at the order of eq. (1), the coupling of the ϕ is easily deduced from that of the ω_8 by simply multiplying the results of the ω_8 by the factor $-\frac{2}{\sqrt{6}}$. With these ingredients and the standard $\gamma\pi\pi$ coupling we can write the contribution of a Feynman diagram with the ϕ going to a photon which then couples to a pair of pions, and which is given by

$$i\,\mathcal{L}_{\phi\pi^+\pi^-} = ie^2 \frac{\sqrt{2}\,F_V}{3\,M_\phi} \epsilon^\mu(\phi)(p_+ - p_-)_\mu\, F(M_\phi^2), \tag{2}$$

where p_+ and p_- are, respectively, the momenta of positive and negative pions and $F(q^2)$ is the pion electromagnetic form factor, which at the ϕ mass is given by $F(M_\phi^2) = -1.56 + i\,0.66$. This can be compared with the coupling of the ϕ to $K^+\,K^-$, or $K^0\,\bar{K}^0$, which can be obtained from the G_V term in Eq. (1) and reads

$$i\,\mathcal{L}_{\phi K^+K^-} = -i\,g_{\phi K^+K^-}\,\epsilon^\mu(\phi)\,(p_+ - p_-)_\mu\,, \quad g_{\phi K^+K^-} = \frac{M_\phi\,G_V}{\sqrt{2}\,f^2}\,, \tag{3}$$

which provides the right ϕ decay width with a value of $G_V = 54.3$ MeV .

By analogy to Eq. (3), Eq. (2) gives a ϕ coupling to $\pi^+\pi^-$

$$g^{(\gamma)}_{\phi\pi^+\pi^-} = -\frac{\sqrt{2}}{3}\,e^2\,\frac{F_V}{M_\phi}\,F(M_\phi^2), \tag{4}$$

which provides the $\phi \to \pi^+\pi^-$ decay width with the tree level photon mechanism. With a value of $F_V = 154$ MeV from the $\rho \to e^+e^-$ decay [13] and using the coupling of Eq. (4) one obtains a branching ratio to the total ϕ width of 1.7×10^{-4}.

In order to evaluate the strong contribution to the process we consider the $K\bar{K} \to \pi^+\pi^-$ amplitude corrected from isospin violation effects due to quark mass differences. The method used is based on the chiral unitary approach to the meson-meson interaction followed in [8, 9]. The technique starts from the $O(p^2)$ and $O(p^4)$ χPT Lagrangian and uses the IAM in coupled channels, generalizing the one channel version of the IAM developed in [7].

Within the coupled channel formalism, the partial wave amplitude is given in the IAM by the matrix equation

$$T = T_2\,[T_2 - T_4]^{-1}\,T_2, \tag{5}$$

where T_2 and T_4 are $O(p^2)$ and $O(p^4)$ χPT partial waves, respectively. In principle T_4 would require a full one-loop calculation, but it was shown in [8] that it can be very well approximated by

$$\mathrm{Re}\, T_4 \simeq T_4^P + T_2\, \mathrm{Re} G\, T_2 \tag{6}$$

where T_4^P is the tree level polynomial contribution coming from the $\mathcal{L}_4$ chiral Lagrangian and G is a diagonal matrix for the loop function of the intermediate two meson propagators which are regularized in [8] by means of a momentum cut-off.

In the present case, in which isospin is broken explicitly and $J = 1$, we are dealing with three two-meson states: K^+K^-, $K^0\bar{K}^0$ and $\pi^+\pi^-$, that we will call 1, 2 and 3, respectively. The amplitude is a 3×3 matrix whose elements are denoted as T_{ij}. The T_2 and T_4^P amplitudes used in the present work and calculated in the isospin breaking case, are collected in the appendix of [14]. The fit of the phase shifts and inelasticities is carried out here in the isospin limit, as done in [8] and there are several sets of L_i coefficients which give rise to equally acceptable fits.

We write in table 1 the values of the coefficients of the different sets of chiral parameters. The corresponding results for the phase shifts and inelasticities can be seen in [14] where it is shown that the small differences in the results appear basically only in the $a_0(980)$ and $\kappa(900)$ resonance regions, where data have also larger errors or are very scarce.

In order to evaluate the contribution to the $\phi \to \pi^+\pi^-$ coupling from the strongly interacting sector we evaluate the $K^+K^- \to K^+K^-$ amplitude (T_{11}) and the $K^+K^- \to \pi^+\pi^-$ amplitude (T_{13}) near the pole of the ω_8 resonance which in our case appears around $M_{\omega_8} = 920$ MeV. Close to the ω_8 pole the amplitudes obtained numerically are then driven by the exchange of an ω_8.

By assuming a coupling of the type of Eq. (3) for the ω_8 to K^+K^- and $\pi^+\pi^-$, these two amplitudes, close to the ω_8 pole, are given by

$$\begin{aligned} T_{11} &= g^2_{\omega_8 K^+K^-} \frac{1}{P^2 - M^2_{\omega_8}}\, 4\, \vec{p}_K \cdot \vec{p}_{K'} \\ T_{13} &= g_{\omega_8 K^+K^-}\, g_{\omega_8 \pi^+\pi^-} \frac{1}{P^2 - M^2_{\omega_8}}\, 4\, \vec{p}_K \cdot \vec{p}_\pi . \end{aligned} \tag{7}$$

where $\vec{p}_i$ is the three-momentum of the i particle in the CM frame.

	$\hat{L}_1$	$\hat{L}_2$	$\hat{L}_3$	$\hat{L}_4$	$\hat{L}_5$	$2\hat{L}_6+\hat{L}_8$	$\hat{L}_7$	q_{max}	$\mathrm{BR}_{\phi\to\pi\pi}$
set 1	0.91	1.61	-3.65	-0.25	1.07	0.58	-0.4	666 MeV	1.3×10^{-4}
set 2	0.91	1.61	-3.65	-0.25	1.07	0.58	0.05	751 MeV	1.0×10^{-4}
set 3	0.88	1.54	-3.66	-0.27	1.09	0.68	0.10	673 MeV	1.3×10^{-4}
	L_1	L_2	L_3	L_4	L_5	$2L_6+L_8$	L_7	μ	
ChPT ref. [15]	0.4 ±0.3	1.4 ±0.3	-3.5 ±1.1	-0.3 ±0.5	1.4 ±0.5	0.5 ±0.3	-0.4 ±0.2	M_ρ	———

Table 1: Different sets of chiral parameters ($\times10^{-3}$) that yield reasonable fits to the meson-meson scattering phase shifts and the corresponding $\phi\to\pi\pi$ branching ratio prediction. We have used a hat to differentiate them from those obtained for standard ChPT. However, as it is explained in [8], we still expect them to be relatively similar once the appropriate scales are chosen (roughly $\mu\simeq 1.2\, q_{max}$, see [8] for details).

By looking at the residues of the amplitudes T_{11}, T_{13} in the ω_8 pole we can get the products $g_{\phi K^+K^-}\, g_{\phi K^+K^-}$ and $g_{\phi K^+K^-}\, g_{\phi\pi^+\pi^-}$. Thus, defining

$$Q_{ij} = \lim_{P^2\to M^2_{\omega_8}} (P^2 - M^2_{\omega_8})\frac{T_{ij}}{4\,\vec{p_i}\cdot\vec{p_j}} \tag{8}$$

we obtain the ratio of the $g_{\phi K^+K^-}$ to $g_{\phi K^+K^-}$ by means of the ratio of Q_{13} to Q_{11}, and hence taking $g_{\phi K^+K^-}$ from Eq. (3), we get the value for $g^{(s)}_{\phi\pi^+\pi^-}$. Then, by adding the above contribution with that of Eq. (4) we can obtain the $\phi\to\pi^+\pi^-$ decay width. We have taken $F_V\, G_V>0$, as demanded by vector meson dominance [13].

Each set of chiral parameters has then been used in the isospin-breaking amplitudes given in the appendix of [14], obtaining a value of $BR(\phi\to\pi\pi)$ given in table 1. The dispersion of the results provides an estimate of the systematic theoretical uncertainties.

From table 1, we obtain, after taking into account the strong contributions

$$Br(\phi\to\pi\pi)_{\text{tree+strong}} \simeq (1.2\pm0.2)\times10^{-4} \tag{9}$$

On the other hand, explicit calculations of the absorptive part of the $\eta\gamma$ intermediate channel [11] give a contribution of about 1/4 of the kaon loops. In order to estimate the uncertainties from neglecting the photonic loops we take a conservative estimate and consider them of the same magnitude as the strong interaction correction, and, hence, add an extra $\pm0.5\times10^{-4}$ uncertainty.

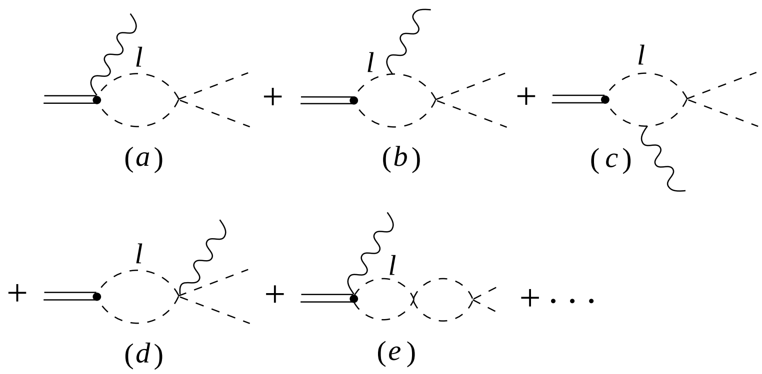

Figure 1: *Diagrams for the decay* $\phi \to \pi^0\pi^0\gamma$.

Adding in quadrature the errors from the different sources, our final result is the band of values:

$$BR(\Phi \to \pi\pi) \simeq 0.7 \text{ to } 1.7 \times 10^{-4}, \tag{10}$$

which is compatible with the present PDG average within errors and lies just between the results of the two recent experiments, which are much more precise, but mutually incompatible.

2 The ϕ radiative decay into $\pi^0\pi^0\gamma$ and $\pi^0\eta\gamma$

The ϕ meson cannot decay into two pions or $\pi^0\eta$ in the isospin limit. The decay into two neutral pions is more strictly forbidden by symmetry and the identity of the two pions. As a consequence the decay of the ϕ into $\pi^0\pi^0\gamma$ and $\pi^0\eta\gamma$ is forbidden at tree level. However, the ϕ decays into two kaons and the processes described can proceed via the loop diagrams depicted in Fig. 1 where the intermediate states in the loops stand for $K\bar{K}$.

The evaluation of the diagrams of Fig. 1 is done in [16]. The terms with G_V of Eq. (2) contribute to all the diagrams in the figure. However, the F_V term of Eq. (2) only contributes to the diagrams containing the contact vertex $\phi \to \gamma K\bar{K}$, like diagrams (a), (e). The idea follows closely the work of [17] but for the treatment of the final state interaction of the mesons one uses here the norperturbative chiral techniques. In this case for L=0, which is the only partial wave needed, one can use the results of [18], where it is proved that the

use of the Bethe Salpeter equation in connection with the lowest order chiral Lagrangian and a suitable cut off in the loops gave a good description of the meson meson scalar sector. Furthermore, in [19] it was proved that the meson meson amplitude in those diagrams factorized on shell. The loops of type (a), (b) and (c) can be summed up using arguments of gauge invariance following the techniques of [19, 20] and lead to a finite amplitude. On the other hand, the terms involving F_V and a remnant momentum dependent term from the G_V Lagrangian in Eq. (2) only appear in the contact vertex $\phi \to \gamma K\bar{K}$, and the diagrams of type (b), (c) are now not present. Hence, in this case the only loop function involved is the one of two mesons which is regularized as in [18] for the problem of the meson meson scattering. The average over polarization of the ϕ for the modulus square of t matrix is then easily written and for the case of $\pi^0\pi^0\gamma$ decay one finds

$$\bar{\sum}\sum |t|^2 = \frac{2}{3}e^2 \left| \frac{M_\phi G_V}{f^2\sqrt{3}} \tilde{G}_{K^+K^-} t^{I=0}_{K\bar{K},\pi\pi} + \frac{K}{f^2\sqrt{3}} \left(\frac{F_V}{2} - G_V \right) G_{K^+K^-} t^{I=0}_{K\bar{K},\pi\pi} \right|^2$$

For the $\phi \to \pi^0\eta\gamma$ case we have

$$\bar{\sum}\sum |t|^2 = \frac{4}{3}e^2 \left| \frac{M_\phi G_V}{f^2\sqrt{2}} \tilde{G}_{K^+K^-} t^{I=1}_{K\bar{K},\pi\eta} + \frac{K}{f^2\sqrt{2}} \left(\frac{F_V}{2} - G_V \right) G_{K^+K^-} t^{I=1}_{K\bar{K},\pi\eta} \right|^2$$

where $\tilde{G}_{K^+K^-}$ and $G_{K^+K^-}$ are the loop functions mentioned above.

We have evaluated the invariant mass distribution for these decay channels and in Fig. 2 we plot the distribution dB/dM_I for $\phi \to \pi^0\pi^0\gamma$ which allows us to see the $\phi \to f_0\gamma$ contribution since the f_0 is the important scalar resonance appearing in the $K^+K^- \to \pi^0\pi^0$ amplitude [18]. The results are obtained using G_V=55 MeV and F_V=165 MeV, which are suited to describe the $K\bar{K}$ and e^+e^- decay of the ϕ. The solid curve shows our prediction, with $F_V G_V > 0$, the sign predicted by vector meson dominance, as we quoted above. The dashed curve is obtained considering $F_V G_V < 0$. In addition we show also the results of the intermediate dot-dashed curve which correspond to taking for G_V and F_V the parameters of the ρ decay, G_V=69 MeV and F_V=154 MeV. We compare our results with the recent ones of the Novosibirsk experiment [21]. We can see that the shape of the spectrum is relatively well reproduced considering statistical and systematic errors (the latter ones not shown in the

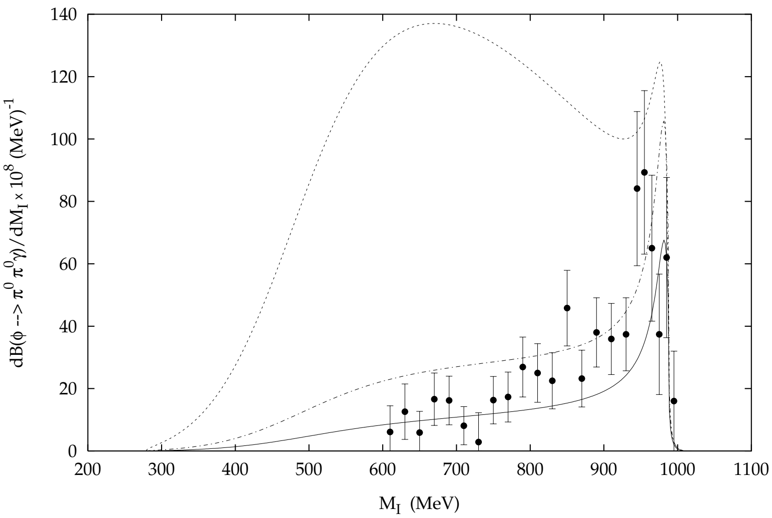

Figure 2: *Distribution dB/dM_I for the decay $\phi \to \pi^0\pi^0\gamma$, with M_I the invariant mass of the $\pi^0\pi^0$ system. Solid line: our prediction, with $F_V G_V > 0$. Dashed line: result taking $F_V G_V < 0$. The data points are from [21] and only statistical errors are shown. The systematic errors are similar to the statistical ones [21]. The intermediate, dot-dashed curve corresponds to the results obtained using the G_V and F_V parameters of the ρ decay.*

figure). The results considering $F_V G_V < 0$ are in complete disagreement with the data.

The finite total branching ratio which we find for the $\phi \to \pi^0\pi^0\gamma$ decay is 0.8×10^{-4}, which is slightly smaller than the result given in [21], $(1.14 \pm 0.10 \pm 0.12) \times 10^{-4}$, where the first error is statistical and the second one systematic. The result given in [22] is $(1.08 \pm 0.17 \pm 0.09) \times 10^{-4}$, compatible with our prediction. Should we use the values for F_V and G_V of the ρ decay we would obtain 1.7×10^{-4}. The branching ratio obtained for the case $\phi \to \pi^0\eta\gamma$ is 0.87×10^{-4}. The results obtained at Novosibirsk are [23] $(0.83 \pm 0.23) \times 10^{-4}$ and [22] $(0.90 \pm 0.24 \pm 0.10) \times 10^{-4}$. Should we use the values for F_V and G_V of the ρ decay we would obtain 1.6×10^{-4}. The spectrum, not shown, is dominated by the a_0 contribution.

The results reported here are two examples of the successful application

of the chiral unitary techniques. A recent review of multiple applications of these methods can be seen in [25].

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